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Life and Reliability Modeling of Bevel Gear Reductions

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SUMMARY

A reliability model is presented for bevel gear reductions with either a single input pinion or dual input pinions of equal size. The dual pinions may or may not have the same power applied for the analysis. The gears may be straddle mounted or supported in a bearing quill. The reliability model is based on the Weibull distribution. The reduction's basic dynamic capacity is defined as the output torque which may be applied for one million output rotations of the bevel gear with a 90 percent probability of reduction survival.

INTRODUCTION

This work describes an analytical computer simulation tool for the design of bevel gear reductions. A model is presented which calculates the reliabilities and resulting statistical lives of reductions composed of bevel gears and their support bearings. The bevel gear reduction may include single or dual pinion input and a variety of bearing types and support locations. The reduction loading is assumed to be composed of pure torque inputs and output. The calculations combine the lives and reliabilities of the individual bearings and gears in the reduction in a strict series reliability model.

This model will simulate various reduction designs at different power levels. The calculations predict relative lives of a reduction at a given reliability. The calculations can also determine the dynamic capacity of the reduction in terms of the dynamic capacities of its components. With this approach, comparisons of complete designs can be made at the design stage.

Experimental testing programs are normally used by industry to evaluate the relative merits of different reductions (refs. 1 to 3). These programs include real time duty cycle simulation and quasi-static overload power tests for accelerated aging of the reductions. These tests provide an important "proof" test for physically checking the validity of any analytical design model used in the paper stage of design. However, they are extremely costly in terms of time and resources. They should be complemented with computer simulations of the many possible reduction designs, so that only nearly optimal designs are brought forward to the testing stage of a reduction's development.

Computer programs are available for Lundberg-Palmgren fatigue life analyses of various bearings and bearing shaft arrangements (refs. 4 and 5). This theory has also been applied in the analysis of fatigue life for spur and helical gear sets (refs. 6 to 10). In two previous papers, the life and reliability of a helicopter planetary reduction have been developed as functions of the lives and reliabilities of its components (refs. 11 and 12).

The models described herein are based on the assumption that the reduction is adequately lubricated and that the components are well designed. For gears, this means that sufficient rim thicknesses and proper materials are used to prevent premature tooth breakage failures. In addition, it is assumed that the tooth form geometry and lubricant have been selected to prevent gear tip scoring. Both of these modes of failure are preventable with adequate design (ref. 13).

However, surface pitting in the full load region of the tooth face is not preventable, due to the lack of a surface fatigue endurance limit for high strength gears (refs. 6 to 10, 14 and 15). As with rolling element bearings, gear teeth will fail eventually in surface pitting even in a well designed, well lubricated reduction, regardless of the loads. Thus, the life and reliability models for the spiral bevel gear reduction are based on the pitting fatigue life and reliability models for the bearings (refs. 4 and 5) and the gears (refs. 6 to 10) in the reduction.

In order to adequately model the bevel reduction, a modular approach is used in which the force and motion analyses of the reduction are separated from the life and reliability analyses. The dynamic capacity models are also separated algebraically from the prior calculations. In this way, the calculations can be performed sequentially and the complexity of the analyzed reduction can be greatly increased.

NOMENCLATURE

A distance from gear to closest bearing, mm

A_o distance from pitch cone center to gear back face, mm

a_b bevel gear tooth addendum, mm

B distance from gear to furthest bearing, mm

B_1 gear material constant, MPa

b length of major axis on gear tooth contact ellipse, mm

C component dynamic capacity, N

D dynamic capacity, N·m

d_b bevel gear tooth dedendum, mm

F bearing force, N

f gear face width, mm

$\mathfrak{L}$ component life, cycles

N number of teeth

P_d diametral pitch, 1/mm

S reliability - probability of survival

T torque, N·m

W gear tooth load, N

Γ gear cone pitch angle, deg

Λ angle of separation of dual pinions, deg

$\Sigma\rho$ curvature sum, 1/mm

Σ gear shaft angle, deg

φ_n normal pressure angle, deg

ψ spiral angle, deg

Subscripts:

a axial

b bearing

e equivalent

g gear

n normal

p pinion

R reduction

r radial

t tooth, for lives and capacities; tangential, for gear load W; thrust, for bearing force F

10 90 percent survival life

Exponents:

e Weibull slope

p load - life factor

REDUCTION DESCRIPTION

The reductions modeled in this analysis are single output reductions composed of a single bevel gear drive with one or two bevel pinion inputs. The reduction is composed of the spiral bevel gears and the bearings which support the pinion and gear shafts.

Figure 1 is a schematic representation of a single input bevel gear reduction. In this figure, A_o is a measure of the bevel size. It is the distance from the apex of the bevel cones to the back surface of the bevel pinion and gear teeth. This distance is measured along the pitch line of the two pitch cones. The distance, f, which is measured along the same pitch line, is the face width of the two gears. The speed ratio is the number of teeth on the gear, N_g, divided by the number of teeth on the pinion, N_p. The cone geometry of the reduction is then established by the shaft angle between the input pinion and output gear shafts. This angle, Σ, is measured in figure 1 in the plane defined by these two shafts.

The two pitch angles, Γ_g and Γ_p, shown in figure 1 for the gear and pinion are directly related to the numbers of teeth on the two gears and the shaft angle, Σ, by equations (1) and (2) (ref. 16).

$$\tan \Gamma_g = \frac{\sin \Sigma}{\left(\frac{N_p}{N_g}\right) + \cos \Sigma} \tag{1}$$

$$\tan \Gamma_p = \frac{\sin \Sigma}{\left(\frac{N_g}{N_p}\right) + \cos \Sigma} \tag{2}$$

The pitch angle of a bevel gear is the half cone angle of the bevel gear. The pitch cone of the bevel gear is the surface which contains the instant centers of the gear for its motion with respect to a meshing gear. These cones correspond to the pitch circles of plane spur gears.

The gears are further defined by the geometry of their meshing teeth. For this analysis, some simplified models are used for the teeth. The input geometry needed to set up these models is shown in figures 2 and 3.

Figure 2 shows a gear in the pitch plane which is tangent to both pitch cones at the line of contact or pitch ray. In this figure, the spiral angle, ψ, is shown as the angle between the pitch ray and a tangent to the circular cutter at the midpoint of the tooth. This angle is positive for a right handed advance of the spiral along the axis of the gear as shown in figure 2. In a spiral bevel gear mesh, the pinion and gear have spiral angles of equal magnitude but of opposite hands. The diametral pitch of the gear and pinion, P_d, is defined at the midpoint of the gear so that average tooth properties are used in the strength and life calculations. The diametral pitch is:

$$P_d = \frac{N_g}{2\left(A_o - \frac{f}{2}\right)\sin \Gamma_g} \tag{3}$$

Figure 3 shows a view of section AA in figure 2. In this figure one can see the normal tooth geometry of the gears in the midplane of the tooth. This figure illustrates the normal pressure angle, φ_n, the addendum, a_b, and the dedendum, d_b, of the bevel teeth in their midplane.

The direction of rotation of a gear is positive for clockwise rotation when looking at the apex of the cone from the gear.

The description of the bevel reduction includes the support bearing locations and types. Figure 4 shows the two bearing configurations treated in this analysis. For the first case shown in figure 4(a), the two bearings straddle the bevel gear. For the second case shown in figure 4(b), the bevel gear is overhung from a two bearing quill. In both cases, distance A is the distance from the midplane of the gear to the bearing closest to the cone apex and distance B is the distance from the midplane of the gear to the second bearing. Distance A is negative for the overhung mounting case of figure 4(b). The thrust bearing and the types and capacities of the bearings must also be known.

When there are two bevel pinion inputs, it is assumed that the two bevel pinions have the same face width, pitch angle and number of teeth. The permissible differences are the torque levels and the shaft locations. In figure 5, the two bevel pinions of a twin input bevel reduction are shown in the plane defined by their two axes. The angle Λ describes the separation of these two pinions in this plane.

In all cases, the material strength or bearing capacity, Weibull exponent, and load - life exponent must be specified for each component or group of similar components. The pinion torques and speed define the reduction loading.

Life and Dynamic Capacity Models

The life and dynamic capacity models of this study consider failure from the fatigue pitting of the bearings and the gear teeth in the reduction. The life model is that of Palmgren's (refs. 17 and 18) for rolling element bearings:

$$L_{10} = \left(\frac{C}{F}\right)^p \tag{4}$$

where L_{10} is the life of the component for a 90 percent probability of survival, F is the applied equivalent load, and C is the basic dynamic capacity of the component. This basic dynamic capacity is the equivalent load for which 90 percent of the components will survive one million load cycles. The exponent, p, is called the load - life factor. Equation (4) is the analytical expression for a load - life diagram in which there is no endurance limit.

This model for the component life as a function of load has been combined with the Weibull distribution for probability of survival as a function of life at a given load for ball (ref. 16) and roller bearings (refs. 17 and 18) and for gear teeth (refs. 6 to 10). The result is a complete description of life

and reliability as a function of load for a component. The two parameter Weibull distribution is:

$$\log\left(\frac{1}{S}\right) = \log\left(\frac{1}{0.9}\right)\left(\frac{\ell}{\ell_{10}}\right)^{e} \tag{5}$$

where S is the reliability (i.e., the probability of survival) at the component life ℓ, ℓ_{10} is the life for a 90 percent probability of survival and e is the Weibull slope.

The reduction life and reliability models of this study combine these models for the components with a strict series probability law that states that the probability of survival of the reduction is the product of the probabilities of survival of the components:

$$S_R = S_a \times S_b \times S_c \ldots \tag{6}$$

This strict series probability law is justified on the basis of the high speed of reduction components and the effects of loose debris. If any component fails, debris may be present in the reduction which could accelerate the wear damage to the reduction. Thus, an overhaul of the reduction is advised to return the reduction to its initial state of high reliability.

To simplify the calculations, the reduction life and dynamic capacity models are obtained in a sequential manner. This sequential approach breaks the analysis down into a series of analysis modules.

The first set of modules determine each component's load from the applied load and the reduction parameters.

The second set of modules determine each component's system life in the common counting base of output shaft rotations. These modules use the component dynamic capacity, component load, and component load frequency to determine the component's system life.

The third set of modules determine each component's system dynamic capacity as an equivalent output torque for which 90 percent of the components will survive one million output rotations. These dynamic capacities differ from the component's individual dynamic capacities in load units and number of load cycles. They are needed to determine the reduction dynamic capacity.

The fourth module determines the reduction life in output shaft rotations from the individual component's system lives.

The fifth module then determines the reduction dynamic capacity from the individual component's system dynamic capacities.

Component Loading

The gear tooth contact force analysis for a spiral bevel gear mesh is illustrated in figure 6. In this figure, the tooth load on a spiral bevel gear is shown as three orthogonal components applied at the center of the

tooth face on the pitch cone. These three components are aligned relative to the axis of this gear alone (ref. 19). They are: a tangential load, W_t, which produces the torque on the gear; an axial load, W_a, which produces the axial load thrust on the gear shaft; and a radial load, W_r. For the output gear, the loads are:

$$W_t = \frac{T_g}{\left(A_o - \frac{f}{2}\right)\sin \Gamma_g} \tag{7}$$

$$W_a = \frac{W_t}{\cos \psi} (\tan \varphi_n \sin \Gamma_g \mp \sin \psi \cos \Gamma_g) \tag{8}$$

and

$$W_r = \frac{W_t}{\cos \psi} (\tan \phi_n \cos \Gamma_g \pm \sin \psi \sin \Gamma_g) \tag{9}$$

where T_g is the torque on the output gear.

In equations (8) and (9), the sign of the last term depends on the spiral hand and direction of gear rotation. The top signs (- in eq. (8) and + in eq. (9)) are valid for a right hand spiral output gear rotating counterclockwise (looking at the gear from the side opposite the apex), or a left hand spiral rotating clockwise. The bottom signs (+ in eq. (8) and - in eq. (9)) are valid for a right hand spiral output gear rotating clockwise or a left hand spiral rotating counterclockwise.

The total resultant load is the normal tooth load on which the gear tooth life and basic dynamic capacity are based:

$$W_n = \left[W_t^2 + W_a^2 + W_r^2\right]^{1/2} \tag{10}$$

For the bearings which support the output gear, the thrust load is equal to the axial gear force.

$$F_t = W_a \tag{11}$$

The radial forces on the two bearings are given by:

$$F_{r_1} = \frac{\left[W_t^2 B^2 + [W_a N_g/(2P_d) - W_r B]^2\right]^{1/2}}{A + B} \tag{12}$$

and:

$$F_{r_2} = \frac{\left[W_t^2 A^2 + [W_a N_g/(2P_d) - W_r A]^2\right]^{1/2}}{A + B} \tag{13}$$

This force analysis is valid for the bearings of an output gear which is loaded by a single pinion.

The forces on the input pinion gear and bearings which support the input pinion gear can also be determined using equations (7) through (13). The gear subscript, g, is replaced by the pinion subscript, p, in these equations. The top signs (- in eq. (8) and + in eq. (9)) are valid for right hand spiral pinion gear rotating clockwise (looking at the pinion from the side opposite the apex), or a left hand spiral rotating counterclockwise. The bottom signs (+ in eq. (8) and - in eq. (9)) are valid for a right hand spiral rotating counterclockwise or a left hand spiral rotating clockwise.

It should be noted that the radial load on the pinion equals the axial load on the gear only for right angle drives. However, the total resultant tooth load on one gear must be equal and opposite to the total resultant tooth load on the mating gear.

The frequency of loading of the components is directly related to the shaft speed of each component. Each gear tooth sees one load cycle per rotation and each bearing sees one load cycle for each shaft rotation in the single mesh bevel reduction. The hours of operation of a component for its given number of load cycles can be found by dividing the number of load cycles by the shaft speed in revolutions per hour (rpm * 60).

For the case of bevel reductions with dual pinion inputs, the loads on the output bevel gear support bearings are the vector summations of the bearing loads caused by the two separate gear forces. An example of these loads is shown in figure 7. In addition, the teeth of the output gear see two load cycles for each rotation at potentially different load levels.

Component Dynamic Capacity

Each bearing and gear in the reduction has a load which will cause 10 percent of a large sample of those components to fail by pitting at or before one million applications of that load. This is the component dynamic capacity.

For bearings, this capacity, C_b, is available from the manufacturer for each particular bearing (ref. 19). The load - life factors for rolling element bearings are normally taken as $p_b = 3.0$ for ball bearings and as $p_b = 3.33$ for cylindrical rolling element bearings.

For gears, this capacity is not tabulated directly for particular gears. The dynamic capacity of a gear tooth is proportional to the Hertzian contact pressure squared for applications in which the major axis of the contact ellipse is significantly larger than the minor axis. With this proportionality, the dynamic capacity of a gear tooth, C_t, can be expressed as:

$$C_t = B_1 \frac{b}{\Sigma\rho} \tag{14}$$

where B_1 is a material constant, b is the length of the major axis of the contact ellipse and $\Sigma\rho$ is the curvature sum in the direction of the minor

axis of the contact ellipse. The material constant, B_1, is the experimental load - stress factor, K_1, of Buckingham (ref. 20).

Equations (14) and (4) can be used to evaluate this material constant for modern materials from recent gear life tests (ref. 9). By assuming that F in equation (4) is the normal tooth load, W_n, and noting that a lubrication life adjustment factor of about 0.5 was used in reporting the results of the laboratory tests in reference 9, the material constant for case-hardened AISI 9310 Vacuum Arc Remelt Steel gears can be determined as $B_1 = 242$ MPa (35 000 psi). The load - life factor for the gear teeth in these tests was $p_g = 4.3$.

Once the basic dynamic capacity and the load - life factor are known for a component, equation (4) can be used to determine the 90 percent reliability life in million load cycles for the applied load on the component.

Component System Lives

To obtain the system life of the single mesh bevel gear, the component life of a single tooth must be converted to the life of the entire gear. This can be done by considering the reliability of the gear as the product of the reliabilities of its teeth using the strict series reliability model of equation (6):

$$S_g = S_t^{N_g} \tag{15}$$

Taking the log of the reciprocal of this relationship yields:

$$\text{Log}\left(\frac{1}{S_g}\right) = N_g \text{ Log}\left(\frac{1}{S_t}\right) \tag{16}$$

Substitution of equation (5) into both sides of this equation and taking the e_g^{th} root yields:

$$\frac{\ell_g}{\ell_{g_{10}}} = (N_g)^{1/e_g} \left(\frac{\ell_t}{\ell_{t_{10}}}\right) \tag{17}$$

The Weibull slope, e_g, for the gears tested in reference 9 is 2.5.

For gears which have a single load, the number of tooth load cycles equals the number of gear load cycles. Equation (17) can then be reduced to:

$$\ell_{g_{10}} = \left(\frac{1}{N_g}\right)^{1/e_g} \left(\frac{C_t}{W_n}\right)^{p_g} \tag{18}$$

by the substitution of the appropriate form of equation (4) for the gear tooth life in terms of the applied load W_n and the tooth dynamic capacity C_t.

Since the gear sees one complete load cycle for each output shaft rotation, this life is in the system life unit of output gear rotations.

A similar analysis can be performed for the system life of a pinion. The pinion life must be converted from pinion rotations to output gear rotations by the gear ratio.

$$\ell_{p_{10}} = \left(\frac{N_p}{N_g}\right)\left(\frac{1}{N_p}\right)^{1/e_g}\left(\frac{C_t}{W_n}\right)^{p_g} \tag{19}$$

The final gear life to be considered is that for the output gear when it has two input loads. The number of tooth load cycles is double the number of gear rotations. Since the two loads may differ, an equivalent load must be used which produces the same fatigue damage as the two separate loads. This equivalent load is a weighted average of the two loads:

$$W_e = \frac{\left(W_{n_1}^{p_g} + W_{n_2}^{p_g}\right)^{1/p_g}}{2} \tag{20}$$

With the two changes of load frequency and magnitude, equation (18) becomes:

$$\ell_{g_{10}} = \left(\frac{1}{2}\right)\left(\frac{1}{N_g}\right)^{1/e_g}\left(\frac{C_t}{W_e}\right)^{p_g} \tag{21}$$

for the gear loaded by two pinions.

The system lives of the bearings are found from their component lives in a similar fashion. For bearings on the output shaft:

$$\ell_{gb_{10}} = \left(\frac{C_b}{F_e}\right)^{p_b} \tag{22}$$

directly, where C_b is the bearing dynamic capacity, p_b is the bearing load - life factor and F_e is the equivalent bearing load. The system lives of the bearings on a pinion shaft are adjusted by the gear ratio to obtain the lives in the proper units of output gear rotations.

$$\ell_{pb_{10}} = \left(\frac{N_p}{N_g}\right)\left(\frac{C_b}{F_e}\right)^{p_b} \tag{23}$$

Component System Dynamic Capacities

The dynamic capacity of each component can now be expressed as an output torque. By taking the p^{th} root of equation (4) and replacing the ratio of component dynamic capacity to component equivalent load by the ratio of compo-

nent system dynamic capacity to reduction output torque, the component system lives can be used to determine the component system dynamic capacities.

$$D_i = \left(\ell_{i_{10}}\right)^{1/p_i} T_g \tag{24}$$

These dynamic capacities are in units of output torque and express the output torque of the reduction at which 90 percent of a set of similar components will survive for one million output rotations.

Reduction Life

The product rule may be now used to express the probability of survival of the entire reduction in terms of the probabilities of survival of its component parts:

$$S_R = \prod_{i=1}^{n} S_i \tag{25}$$

The probability distribution for the survival of the entire reduction can be obtained by substituting the relations of equation (5) for each component 90 percent reliability life, $\ell_{i_{10}}$, into the natural log of the reciprocal of equation (25):

$$\log\left(\frac{1}{S_R}\right) = \log\left(\frac{1}{0.9}\right) \sum_{i=1}^{n} \left(\frac{\ell_R}{\ell_{i_{10}}}\right)^{e_i} \tag{26}$$

In this equation, ℓ_R is the life of each component and of the entire reduction for a reduction reliability of S_R.

This relation is not a strict Weibull relationship between system life and system reliability. The equation would represent a true Weibull distribution only if all the Weibull exponents, e_i, were equal, which is not generally true. The relationship can be solved for reliability, S_R, as a function of reduction life, ℓ_R, and plotted on Weibull coordinates.

This plot of percent probability of failure versus reduction life can be approximated quite reasonably by a straight line. This straight line approximation can be obtained with a linear regression in Weibull coordinates over the range $0.5 < S_R < 0.95$. The slope of this straight line approximation is the reduction Weibull slope e_R and the reduction life at which the straight line approximation indicates a reliability of $S_R = 0.9$ is the reduction 90 percent reliability life, ℓ_{R10}, at the given load, T_g. The equation for this fitted Weibull relation is:

$$\log\left(\frac{1}{S_R}\right) = \log\left(\frac{1}{0.9}\right)\left(\frac{\ell_R}{\ell_{R_{10}}}\right)^{e_R} \tag{27}$$

Reduction Dynamic Capacity

The basic dynamic capacity for the reduction, D_R, is the output torque which will result in a 90 percent reliability life for the reduction of one million output shaft rotations. By letting $S_R = 0.9$ in equation (26) and $\ell_R = 1$ (million output rotations) and substituting the various component system life to component system dynamic capacity relations in the form of equation (24), one has:

$$1 = \sum_{i=1}^{n} \left(\frac{D_R}{D_i}\right)^{e_i p_i} \tag{28}$$

where each D_i is the basic dynamic capacity of a single reduction component. This equation can be solved for D_R by iteration.

The approach used here is to determine a series of 90 percent reliability lives for the reduction using equation (26) and a series of different output torques. A log-log plot of output reduction torque versus output shaft rotations is then generated from this data. The slope of this curve is the negative of the reciprocal of the load life exponent p_R for the reduction. The initial estimate of the dynamic capacity of the reduction, D_R, is the value of the output torque on the curve which corresponds to a life of one million output rotations.

The approximate load life curve is obtained using a linear regression in this log-log plane over a range of output torques from one tenth the dynamic capacity to the dynamic capacity of the reduction. With this approximation, the load life relation for the reduction is given by:

$$\ell_{R_{10}} = \left(\frac{D_R}{T_g}\right)^{p_R} \tag{29}$$

As for the Weibull curve, the load life exponent, p_R, is taken from the slope of the linear regression and the reduction dynamic capacity, D_R, is the output torque on the regression line which corresponds to a life of one million output rotations.

Example

Consider a single reduction of 146 kW at an output speed of 258 rpm. The shaft angle between the pinion and gear is 81.8° and much of the geometry, loading and capacity information is listed in table I.

For this example, the diametral pitch of the gears is $P_d = 0.189$ mm^{-1}, the cone distance is $A_o = 260$ mm, and the gear face width is $f = 65$ mm. The spiral angle is $\psi = 25°$ and the normal pressure angle is $\varphi_n = 20°$. The pinion is a left handed spiral driving in a counterclockwise direction.

As indicated in table I, the load on the gear teeth has negative components for the axial load on the pinion and the radial load on the gear. These two components are opposite the expected directions shown in figure 6. The negative axial load on the pinion is toward the cone center and the negative

radial load on the gear is outward. The negative directions for these components are a result of the combination of the high cone angle on the gear and the spiral angle.

In the example, both gears are straddle mounted, with small thin race bearings. The thrust bearing on the gear is the one closest to the cone center. The bearings are tapered and straight rolling element bearings with Weibull exponents of 1.5 and load - life factors of 3.3. The bearing dynamic capacities are given as C_b in the table.

For the gears, the curvature sum at the pitch point in the center of the gear face is 0.068 mm^{-1}. This was calculated using a Tregold Approximation (ref. 20), which reduces a bevel gear problem to a spur gear problem. The length of the major axis of the contact ellipse is taken as one-half the gear face width or 32.5 mm. For this geometry and AISI 9310 Vacuum Arc Remelt Steel gears, the gear tooth dynamic capacity is 115,700 N for both gear and pinion. The Weibull exponent is 2.5 and the load - life factor is 4.3.

The Weibull plot of percent probability of overhaul versus life in million output rotations is given for the reduction in figure 8. This plot is very close to a straight line with a system 90 percent reliability life for the reduction of 16.2 million output rotations and a Weibull slope of 2.03. This system life is dominated by a combination of the pinion life and the weakest bearing life.

Figure 9 is a plot of the Weibull failure distribution function for the weakest bearing - the inside pinion bearing which has a 90 percent reliability life of 28.6 million output rotations. Figure 10 is a plot of the Weibull failure distribution function for the pinion with its 90 percent reliability life of 23 million output rotations. Figure 11 is the plot of the Weibull overhaul distribution function for the reduction. For this design, the model predicts that most overhauls would be a direct result of pitting fatigue failures of the pinion gear and the pinion bearings.

The system dynamic capacities of the components are listed in the table. Figure 12 is a log - log plot of reduction output torque versus reduction life in million output rotations for this example. The dynamic capacity of the reduction is found to be 11,060 N·m with a load - life exponent of 3.72.

CONCLUSIONS

A reliability model for a bevel gear reduction has been derived. The reliability model is based on the reliability models of the bearing and gear components of the reduction. These models are two parameter Weibull distributions of reliability as a function of life. The reduction's 90 percent reliability life and basic dynamic capacity are presented in terms of output shaft rotations.

Due to the different Weibull distributions for the bearing and gear components, the Weibull model for the reduction is an approximate model. In this model, the reduction's 90 percent reliability life, Weibull exponent, basic dynamic capacity and load life exponent are presented.

The calculations for these properties are set up in a modular form to enable the model to fit a large variety of reductions.

This model extends the Weibull reliability theory to bevel reductions. It allows different design configurations to be compared easily from a standpoint of life at the design stage. Finally, it provides a framework for more exact life analyses of these reductions. These analyses can include more exact modeling of the effects of the spiral bevel gear contact geometry and Hertzian stress loading and of compliances in the reduction on the reduction life.

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TABLE I. - COMPONENT LOADING DATA

	Gear	Pinion
N	81	17
Γ, deg	70.4	11.4
rpm	258	1 230
T, N·m	5 384	1 130
W_t, N	25 160	25 160
W_a, N	13 440	-9 500
W_r, N	-7 675	12 220
W_n, N	29 540	29 540
L_{10} (10^6 cycles)	58.7	23
D_1, N·m	13 890	11 170

	Gear Bearings		Pinion bearings	
	A	B	A	B
A or B, mm	81.3	146.1	53.3	71.1
C_b, N	202 400	121 400	78 730	96 480
F_t, N	13 400	0.0	0.0	9 500
F_r, N	23 900	13 400	17 750	10 940
F_e, N	27 000	13 400	17 750	16 720
L_{10} (10^6 cycles)	766	1 451	28.6	68.2
D_1, N·m	40 290	48 890	14 880	19 350

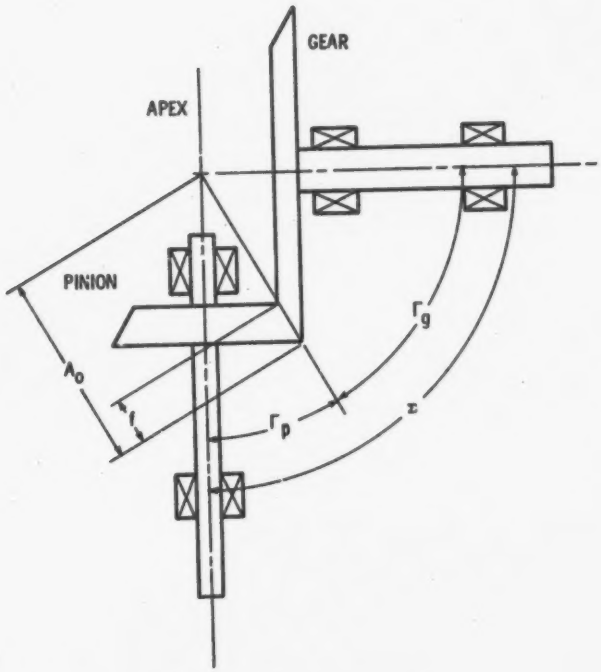


Figure 1. - Pitch cone geometry.

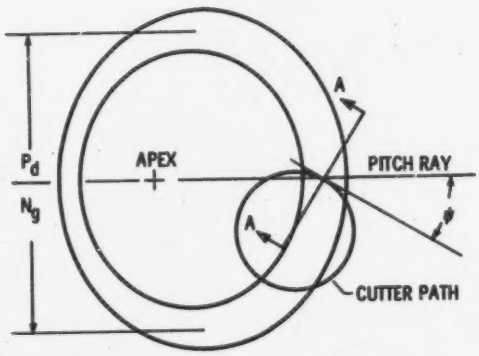


Figure 2. - Spiral Angle

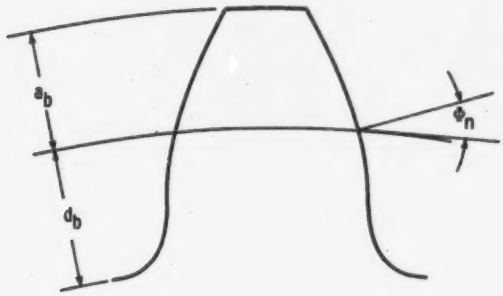
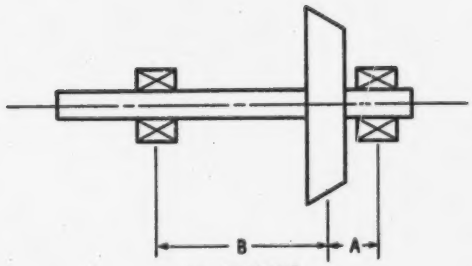
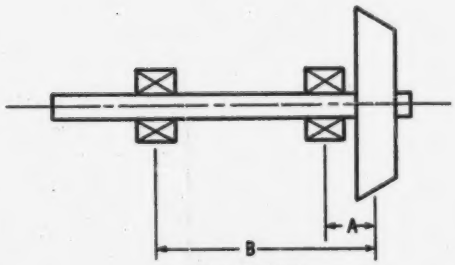


Figure 3. - Normal tooth proportions.



(a) STRADDLE



(b) OVERHUNG

Figure 4. - Bearing support configurations

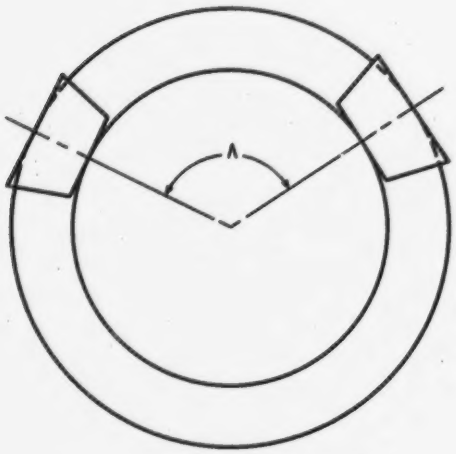


Figure 5. - Dual input bevel gear geometry.

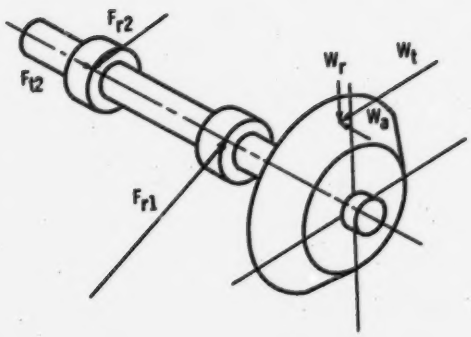


Figure 6. - Bevel gear forces.

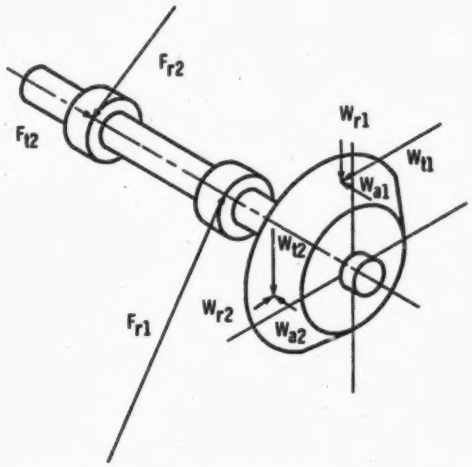


Figure 7. - Dual input bevel gear forces on output gear and bearings.

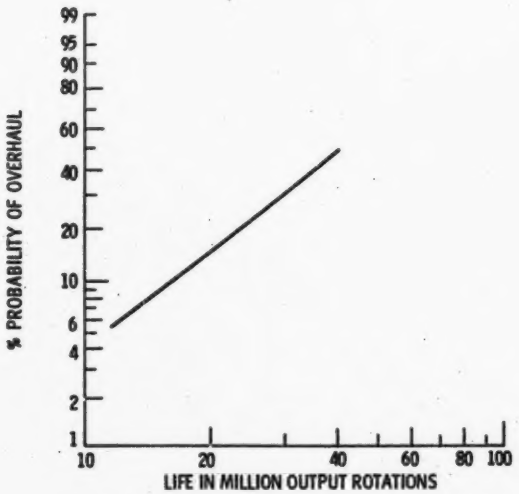


Figure 8. - Bevel reduction reliability versus life.

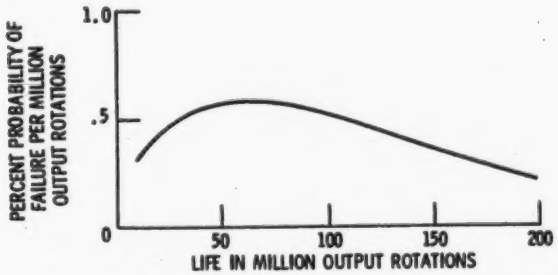


Figure 9. - Pinion radial bearing life distribution.

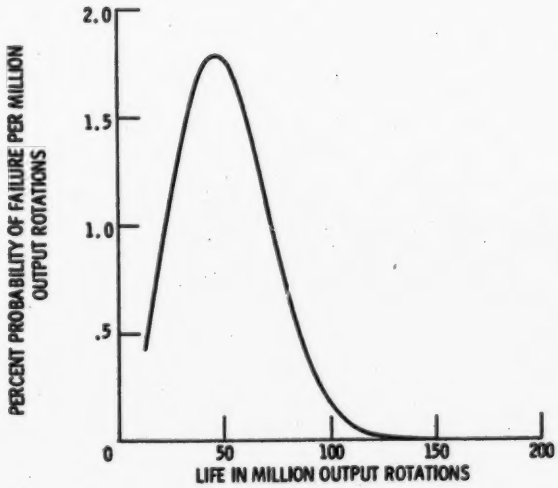


Figure 10. - Pinion life distribution.

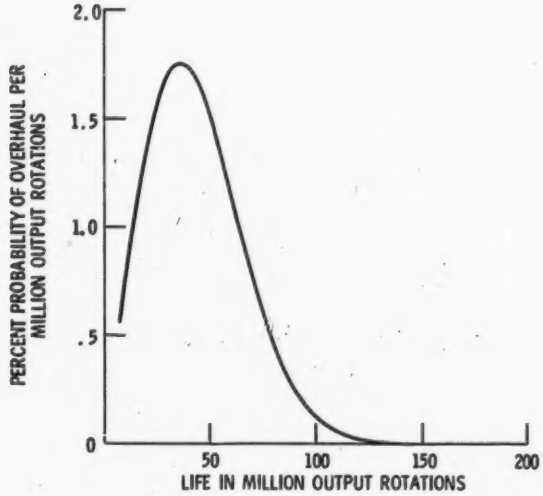


Figure 11. - Bevel reduction life distribution.

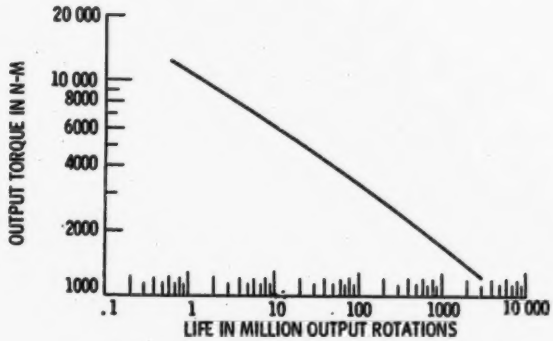


Figure 12. - Bevel reduction output torque versus life.

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15. Supplementary Notes

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16. Abstract

A reliability model is presented for bevel gear reductions with either a single input pinion or dual input pinions of equal size. The dual pinions may or may not have the same power applied for the analysis. The gears may be straddle mounted or supported in a bearing quill. The reliability model is based on the Weibull distribution. The reduction's basic dynamic capacity is defined as the output torque which may be applied for one million output rotations of the bevel gear with a 90 percent probability of reduction survival.

17. Key Words (Suggested by Author(s))	18. Distribution Statement		
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